$$f(x) = (x^3 - 2x + 1)(3 + 5x - 6x^{1/2})$$

(a) $f'(x) = (x^3-2x+1)(5-3x^{-1/2}) + (3x^2-2)(3+5x-6x^{1/2})$

$= (x^3-2x+1)\left(5 - \frac{3}{\sqrt{x}}\right) + (3x^2-2)(3+5x-6\sqrt{x})$

Derivative Rules: Sum, Coef. Rules and

$$\frac{d}{dx}(x^n) = n\,x^{n-1}.$$

$$\frac{d}{dx}(f(x)g(x)) = f(x)g'(x) + f'(x)g(x)$$

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$$

(b) "HEIGHT AT $x=1$" $= f(1)$

$= ((1)^3 - 2(1)+1)(3+5(1)-6\sqrt{1})$

$= (1-2+1)(3+5-6)$

$= 0 \cdot 2 = 0$

(c) "SLOPE AT $x=1$" $= f'(1)$

$= ((1)^3-2(1)+1)\left(5-\frac{3}{\sqrt{1}}\right) + (3(1)^2-2)(3+5(1)-6\sqrt{1})$

$= (0)\cdot(2) + (1)(2) = 2$

Entry Task: Consider the function

$$f(x) = (x^3 - 2x + 1)(3 + 5x - 6\sqrt{x})$$

a. Find $f'(x)$.

b. Find the *height* at x = 1.

c. Find the *slope* at x = 1.

d. Give the *equation of the tangent* line at x = 1.

(d)

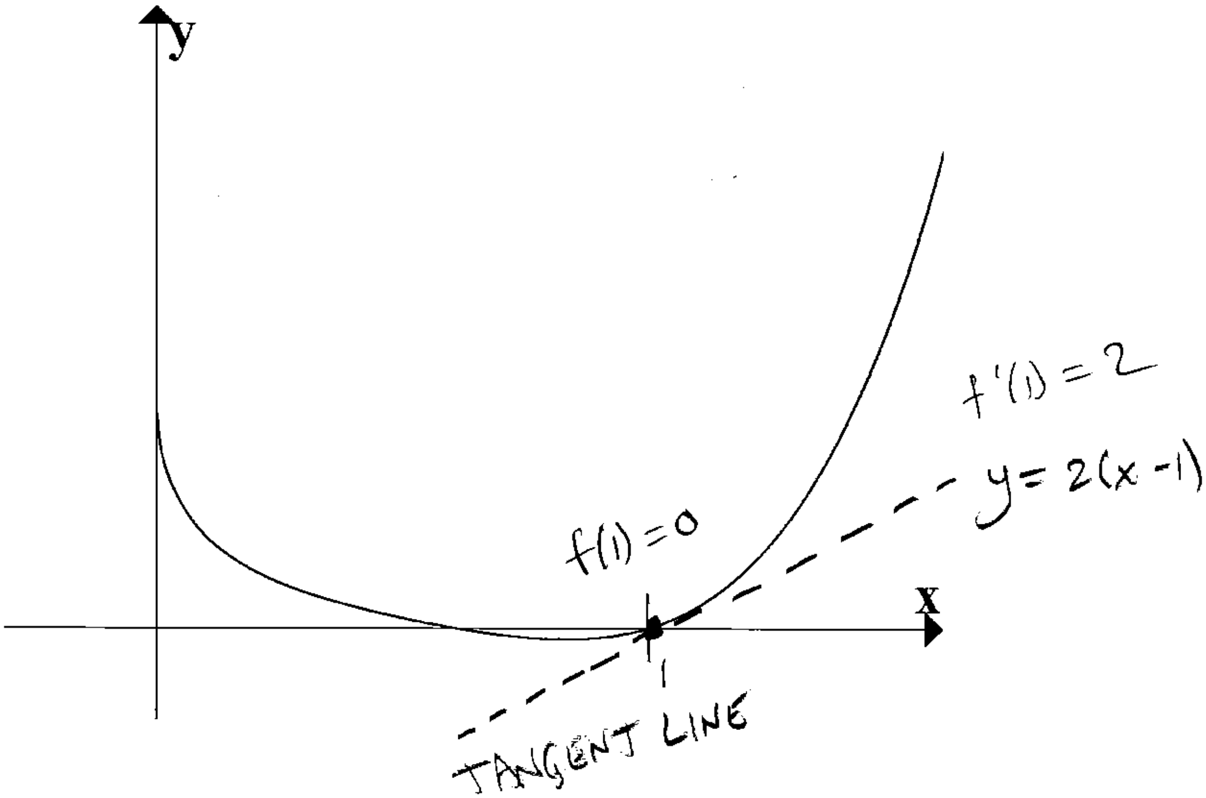
slope x-coord height

$$\boxed{y = 2(x-1) + 0}$$

or

$y = 2x - 2$

$$f(x) = (x^3 - 2x + 1)\left(3 + 5x - 6\sqrt{x}\right)$$



9.7 Chain Rule / Combining Rules

Generalized Power rule:

$$\frac{d}{dx}\left(\left(g(x)\right)^n\right) = n\left(g(x)\right)^{n-1} g'(x)$$

CHAIN RULE:

$$\frac{d}{dx}(f(g(x))) = f'\left(g(x)\right)g'(x)$$

Ex] $y = (3x^2-4x)^{10}$

$u = g(x) = 3x^2-4x$ = INSIDE $\longrightarrow g'(x) = 6x-4$

$f(u) = u^{10}$ = OUTSIDE $\longrightarrow f'(u) = 10u^9$

$y' = 10u^9 \cdot (6x-4) = 10(3x^2-4x)^9 \cdot (6x-4)$

Ex] $y = \sqrt{5x^2+x^3} = (5x^2+x^3)^{1/2}$

$y' = \frac{1}{2}(5x^2+x^3)^{-1/2} \cdot (10x+3x^2) = \frac{10x+3x^2}{2\sqrt{5x^2+x^3}}$

Ex] $y = \frac{10}{(x^5+2x^4)^6} = 10(x^5+2x^4)^{-6}$

$y' = -60(x^5+2x^4)^{-7}(5x^4+8x^3) = \frac{-60(5x^4+8x^3)}{(x^5+2x^4)^7}$

Combining Rules

Step 0: Simplify and rewrite powers.

$$\frac{1}{x^r} = x^{-r}, \quad \sqrt[n]{x} = x^{1/n}$$

Step 1: Identify *overall* form

Sum: $A + B$ $\qquad \overbrace{(x^2+1)^3}^{A} + \overbrace{5x}^{B}$

Product: $F \cdot S$ $\qquad \underbrace{x^5}_{F}\,\underbrace{(x^3+1)^{10}}_{S}$

Quotient: $\frac{N}{D}$ $\qquad \frac{\sqrt{x}}{x^2+3} \begin{array}{l} \leftarrow N \\ \leftarrow D \end{array}$

Chain: $(\text{inside})^n$ $\qquad (x^7+x^5)^{11} \leftarrow (\text{inside})^{11}$

Step 2: Apply rule.

As part of that rule, you likely will have to do more derivative. For those derivatives go to step 1.

Practice: Find the derivatives

Several of these are directly from HW!

1. $y = \dfrac{5\sqrt{1-x^3}}{9}$

$$y = \frac{5}{9}\underbrace{(1-x^3)}_{\text{(inside)}}^{1/2}$$

$$y' = \frac{5}{9}\cdot\frac{1}{2}(1-x^3)^{-1/2}\cdot(-3x^2)$$

$$= -\frac{5}{9}\cdot\frac{3}{2}\cdot\frac{x^2}{\sqrt{1-x^3}} = \boxed{-\frac{5}{6}\cdot\frac{x^2}{\sqrt{1-x^3}}}$$

2. $y = \dfrac{(x^2+2)^3}{x^4+5x}$ $\begin{array}{l}\leftarrow N\\ \leftarrow D\end{array}$

$$N = (x^2+2)^3 \longrightarrow N' = 3(x^2+2)^2(2x)$$

$$D = x^4+5x \longrightarrow D' = 4x^3+5$$

$$\frac{DN' - ND'}{D^2} = \boxed{\frac{(x^4+5x)\cdot 3(x^2+2)^2(2x) - (x^2+2)^3(4x^3+5)}{(x^4+5x)^2}}$$

$$= \frac{(x^2+2)^2\left[6x(x^4+5x) - (x^2+2)(4x^3+5)\right]}{(x^4+5x)^2}$$

3. $y = \left(\frac{2x-4}{x^3+1}\right)^5$

4. $y = \frac{5}{x^3} + 6x^2\sqrt{x^5+1}$

$$y = 5x^{-3} + \underbrace{6x^2}_{F}\underbrace{(x^5+1)^{1/2}}_{S}$$

$$y' = 5\left(\frac{2x-4}{x^3+1}\right)^4 \cdot \left(\frac{(x^3+1)(2) - (2x-4)(3x^2)}{(x^3+1)^2}\right)$$

$$\boxed{y' = -15x^{-4} + 6x^2 \tfrac{1}{2}(x^5+1)^{-1/2}(5x^4) + 12x(x^5+1)^{1/2}}$$

$$\frac{-15}{x^4} \qquad \frac{1}{\sqrt{x^5+1}}$$

5. $y = \dfrac{7}{2(x^4+8)^5} - 5x + 4$

$y = \frac{7}{2}(x^4+8)^{-5} - 5x + 4$

$$\boxed{y' = -\frac{35}{2}(x^4+8)^{-6} \, 4x^3 - 5}$$

6. $y = \underbrace{(t^2+4)^5}_{F}\underbrace{(t^3-2)^4}_{S}$

$$\boxed{y' = (t^2+4)^5 \cdot 4(t^3-2)^3 \cdot 3t^2 + 5(t^2+4)^4(2t)(t^3-2)^4}$$

9.8 Second Derivative

The *second derivative* is the derivative of the derivative. We denote it

$$f''(x) \quad \text{or} \quad \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2}$$

The second derivative represents the rate at which the *rate* of the original quantity is changing.
(*rate of change of rate of change*)

We will interpret what this means more later, for now compute it.

Example:

$$y = x^3 + 2x$$

$$y' = 3x^2 + 2$$

$$\boxed{y'' = 6x}$$

Ex)

$$y = (x^3+2)^4$$

$$y' = 4(x^3+2)^3 \cdot 3x^2 = \underbrace{12x^2}_{F}\underbrace{(x^3+2)^3}_{S}$$

$$\boxed{y'' = 12x^2 \cdot 3(x^3+2)^2 \cdot 3x^2 + 24x(x^3+2)^3}$$

Example: Suppose

$$R(x) = 70x + 0.4x^3$$

is the revenue (in dollars) if you sell x items.

1. What is the marginal revenue (denoted *MR* or $\overline{MR}$) when you sell 10[redacted] items?

Change to 10 in lecture

$MR(x) = R'(x) = 70 + 1.2x^2$

$R'(10) = 70 + 1.2(10)^2$

$= 70 + 1.2(100)$

$= 70 + 120$

$= 190$ $/item

(Next item will bring in about $190 in revenue)

2. What is the rate of change of marginal revenue when you sell 10[redacted] items?

$MR'(x) = R''(x)$

$= 2.4x$

$MR'(10) = 2.4(10) = 24 \quad \frac{\$/\text{item}}{\text{item}}$

MR will go up about 24 $/item over the next item.

MR is increasing
R is "speeding up"
(accelerating)